Generalized Net Model of Multicriteria Decision Making Procedure Using Intercriteria Analysis

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Abstract. The Generalized Nets (GNs) are extensions of the ordinary Petri nets and the other Petri net modifications. A GN-model of a multi-expert multi-criteria decision making process is described. It is extended with an intercriteria analysis of the criteria used by experts – an addition to the standard decision making procedure that changes in the end of a concrete procedure the criteria used by experts during it, so, in the next procedure they work with the modified set of criteria.

Keywords: Decision making · Generalized net · Intercriteria analysis

1 Introduction

In a series of papers of G. Pasi, R. Yager and the author, different multi-criteria decision making procedures are described, that contains essentially new ideas in this area. Here, we describe the process of functioning and the results of the work of such procedures. They are described by one of Petri Net extensions, called Generalized Net (GN; see [1–3]). Each GN has transitions, but now, they contain not only input and output places, but also, moments of activation, duration of the active state, predicates determining which token from input place to which output places can be transfered, capacities of transition arcs and a condition for activation of the transition, when the activation moment arises. Each token has initial and current characteristics, that it can keep and use during the GN functioning.

Below, we use the following three types of sets:

- $E = \{E_1, E_2, ..., E_m\}$ is the set of the measurement tools employed in the decision process;
- $A = \{A_1, A_2, ..., A_p\}$ is the set of the alternatives considered;

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– $C = \{C_1, C_2, ..., C_q\}$ is the set of the criteria used for evaluating the alternatives, which are ordered before their use.

The experts use as standard or given to them criteria, as well as new criteria suggested by the experts, participating in the current procedure. Each expert can use only those of the criteria, that he/she prefers. Each expert has own score in the form of Intuitionistic Fuzzy Pairs (IFPs; see [4,7]) $\langle a, b \rangle$, where $a, b \in [0, 1]$, so that $a + b \leq 1$ and a, b are respectively degree of validity, correctness, etc. and degree of non-validity, non-correctness, etc.

To illustrate the expert's reliability score we give the following example: a sports commentator made 10 prognoses for the results of 10 football matches. In 5 of the cases he predicted correctly the winner, in 3 of the cases he failed and in the remaining 2 cases he did not engage with final opinion about the result. That is why we determine his reliability score as $\langle 0.5, 0.3 \rangle$.

When the i-th expert determines the criteria, which he/she likes to use, he orders them on the vertices of an oriented graph. As it is shown, e.g., in [5], each graph can be represent by an Index Matrix (IM, see [5]) in the form

$$[K, L, \{a_{k_i,l_j}\}] \equiv \begin{array}{c|cccc} & l_1 & l_2 & \ldots & l_n \\ \hline k_1 & a_{k_1,l_1} & a_{k_1,l_2} & \ldots & a_{k_1,l_n} \\ k_2 & a_{k_2,l_1} & a_{k_2,l_2} & \ldots & a_{k_2,l_n} \\ \vdots & & & & \\ k_m & a_{k_m,l_1} & a_{k_m,l_2} & \ldots & a_{k_m,l_n} \end{array},$$

where for a fixed set of indices I and for set $\mathcal{R}$ of numbers (0 and 1; natural, real, etc.), propositions, variables, predicates, IFPs, etc., $K = \{k_1, k_2, ..., k_m\} \subset I, L = \{l_1, l_2, ..., l_n\} \subset I$; for $1 \leq i \leq m$, and for $1 \leq j \leq n : a_{k_i,l_j} \in \mathcal{R}$.

For two IMs different operations are defined, such as "addition", some types of "multiplication", "subtraction", "projection", "restriction", "substitution" and others; and some operators are defined, e.g., hierarchical operators.

When the elements of R are IFPs, the IM is Intuitionistic Fuzzy IM (IFIM). When some IFPs are associated to the arcs and/or vertices of a given graph, the graph becomes an Intuitionistic Fuzzy Graph (IFG).

2 Generalized Net Model

The present GN model (see Fig. 1) is an extension and modification of the model from [10], that is an extension of the GN-model from [9]. Since some of the places and transitions in the new model coincide in both models, we use the same notation. By this reason we use notation for transitions and places: $Y_1, Y_2, Y_3, Y_4, k_1, ..., k_8$ from the second model, $Z_1, Z_2, Z_3, l_1, ..., l_{14}$ from the first model and now for the new model: $X_1, X_2, X_3, X_4, m_1, ..., m_7$.

The GN that we describe below has seven types of tokens – α-, β-, γ-, δ-, ε, ζ- and η-tokens. The third, fourth, ..., and seventh tokens are unique, while α-tokens are m in number and they generate m in number β-tokens, that in place k_7 are united in one token β.

The α-tokens are ordered by some criterion (e.g., alphabetically following experts' names) and their order is not important. Each of the α-tokens has an initial characteristic: *"expert's name, his/her own (current) reliability score $\langle \delta_i, \varepsilon_i \rangle \in [0,1]^2$ such that $\delta_i + \varepsilon_i \leq 1$, and his/her own (current) number of participations in experts' investigations γ_i"* $(1 \leq i \leq m)$.

In the initial time-moment of the GN functioning, the first α-token, α_1 (let α_i denote the i-th α-token) and the γ-token enter places k_1 and l_3, respectively. The later token has initial characteristics *"list of the alternatives, i.e. $A_1, A_2, \ldots, A_p$"*.

The first GN-transition (as we noted above, it is not met in the GN from [10]) has the form:

$$Y_1 = \langle \{k_1\}, \{k_2, k_3\}, \begin{array}{c|cc} & k_2 & k_3 \\ \hline k_1 & true & true \end{array} \rangle.$$

The current token α_i splits into two tokens - the same token α_i that enters place k_2 without a new characteristic and the token β_i that enters place k_3 with a characteristic *"list of the estimation criteria that the i-th expert likes to use for his/her estimation, i.e., $C_{i,1}, C_{i,2}, \ldots, C_{i,q_i}$"*. $(1 \leq i \leq m$ and $1 \leq q_i \leq q_{cu})$, where q_{cu} is the current number of criteria that the experts can use.

$$Y_2 = \langle \{k_2, k_4\}, \{k_4, k_5\}, \begin{array}{c|cc} & k_4 & k_5 \\ \hline k_2 & true & false \\ k_4 & V_{4,4} & V_{4,5} \end{array} \rangle,$$

where
$V_{4,4} =$ "in the current step no token enters place k_1 and the current token has stayed the longest time in place k_4 in respect to all other tokens currently staying in the same place",
$V_{4,5} = \neg V_{4,4}$.

The α-tokens enter place k_4 without a new characteristic. They are collected there, waiting for the beginning of the process of experts' estimation. When predicate $V_{4,5}$ is valid, α-tokens enter place k_5 with the characteristic *"IFG G_i of the expert's opinion for the order among the criteria"*. The vertices of the IFG G_i represent all or a part of the criteria from the last β-token characteristic. The arcs of this IFG are labeled by the i-th expert's scores.

Token ε permanently stays in place k_7, with an initial and current characteristic *"list of actual criteria (q_{cu} in number) that can be used for the current expertise"*.

$$Y_3 = \langle \{k_3, k_7, k_{15}\}, \{k_6, k_7\}, \begin{array}{c|cc} & k_6 & k_7 \\ \hline k_3 & false & true \\ k_7 & V_{7,6} & V_{7,7} \\ k_{15} & false & true \end{array} \rangle,$$

where
$V_{7,6} =$ "in the current step no token enters place k_1",
$V_{7,7} = \neg V_{7,6}$.

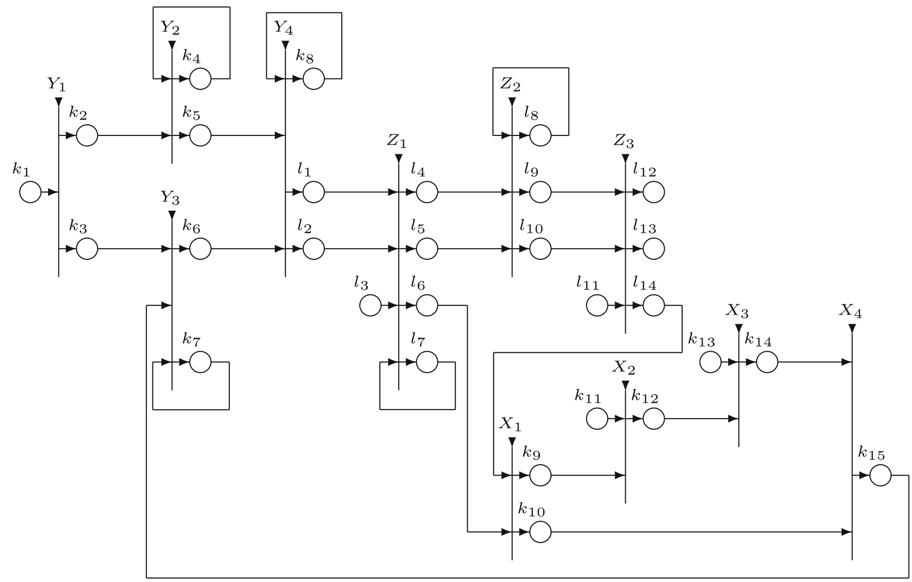

Fig. 1. GN-model

The i-th β-token (let us call it β_i) enters place k_7. It unites with the ε-token, that stays in place k_7. Let us call it β. It waits for the beginning of the process of experts' estimation and it obtains as a current characteristic

$$x_{cu}^{\beta} = x_{cu-1}^{\varepsilon} \cup x_{cu-1}^{\beta_i},$$

i.e., the list of all criteria that are formulated by the first i experts.

Token β from place k_{15} enters place k_7, where it is united with token ε, that obtain as a new (current) characteristic

$$x_{cu}^{\varepsilon} = x_{cu-1}^{\varepsilon} \cup x_{cu}^{\beta}.$$

When predicate $V_{7,6} = true$, then token ε splits into token ε that continues to stay in place k_7 and token β that enters place k_6 with the characteristic $x_{cu-1}^{\beta_i} \cup \lambda_i$, where $\lambda_i \subseteq x_{cu}^{\varepsilon}$ is the list of the criteria, different from these in the set $x_{cu-1}^{\beta_i}$ that the i-th expert likes and will use, too.

$$Y_4 = \langle \{k_5, k_6, k_8\}, \{k_8, l_1, l_2\}, \begin{array}{c|ccc} & k_8 & l_1 & l_2 \\ \hline k_5 & true & false & false \\ k_6 & false & false & V_{6,2} \\ k_8 & V_{8,8} & V_{8,1} & false \end{array} \rangle,$$

where
$V_{6,2} = V_{8,1} =$ "in the current step no token enters place k_5",
$V_{8,8} = \neg V_{8,1}$.

The α-tokens enter place l_1 without any new characteristic, while token β enters place l_2 with a characteristic *"IFG G, obtained by the procedure described below".*

The new IFG G is obtained by operation "$+_\circ$" over the IFGs G_i, which for two IMs $A = [K, L, \{a_{k_i,l_j}\}]$ and $B = [P, Q, \{b_{p_r,q_s}\}]$ has the form

$$A +_\circ B = [K \cup P, L \cup Q, \{c_{t_u,v_w}\}],$$

where

$$c_{t_u,v_w} = \begin{cases} a_{k_i,l_j}, & \text{if } t_u = k_i \in K \text{ and } v_w = l_j \in L - Q \\ & \text{or } t_u = k_i \in K - P \text{ and } v_w = l_j \in L; \\ b_{p_r,q_s}, & \text{if } t_u = p_r \in P \text{ and } v_w = q_s \in Q - L \\ & \text{or } t_u = p_r \in P - K \text{ and } v_w = q_s \in Q; \\ a_{k_i,l_j} \circ b_{p_r,q_s}, & \text{if } t_u = k_i = p_r \in K \cap P \\ & \text{and } v_w = l_j = q_s \in L \cap Q \\ 0, & \text{otherwise} \end{cases}$$

and for two IFPs $\langle a, b\rangle$ and $\langle c, d\rangle$, operation $\circ$ can be, e.g.

$$\langle a, b\rangle \circ \langle c, d\rangle = \begin{cases} \langle \max(a, c), \min(b, d)\rangle, & \text{if } \circ \text{ is } \vee \\ \langle \min(a, c), \max(b, d)\rangle, & \text{if } \circ \text{ is } \wedge \\ \langle \frac{a+c}{2}, \frac{b+d}{2}\rangle, & \text{if } \circ \text{ is } @ \end{cases}$$

or others.

With this operation, we obtain an IM corresponding to the IFG G of all experts' opinions about the criteria ordering. Now, its arcs have intuitionistic fuzzy weights being the disjunctions of the weights, of the same arcs in the separate IFGs. Of course, the new graph may not be well ordered, while the expert graphs are well ordered. Now, we reconfigure IFG G as follows. If there is a cycle between two vertices V_1 and V_2, i.e., there are vertices $U_1, U_2, ..., U_u$ and vertices $W_1, W_2, ..., W_w$, such that $V_1, U_1, U_2, ..., U_u, V_2$ and $V_2, W_1, W_2, ..., W_w, V_1$ are simple paths in the graph, then we calculate the weights of both paths as conjunctions of the weights of the arcs which take part in the respective paths. The path that has smaller weight must be cut in two, removing its arc with smallest weight. If both arcs have equal weights, these arcs will be removed. Therefore, the new graph is already cycle-free. Now, we can determine the priorities of the vertices of the IFG, i.e., the priorities of the criteria. Let them be $\varphi_1, \varphi_2, ..., \varphi_{q_{cu}}$. For example, they can have values $\frac{s-1}{t}$ for the vertices from the s-th level bottom-up of the IFG with $t+1$ levels. We shall use these values below.

The first transition of the GN from [9] that now is in the subnet of the present GN, has the form:

$$Z_1 = \langle \{l_1, l_2, l_3, l_7\}, \{l_4, l_5, l_6, l_7\},$$

$$\begin{array}{c|cccc} & l_4 & l_5 & l_6 & l_7 \\ \hline l_1 & true & false & false & false \\ l_2 & false & false & W_{2,6} & false \\ l_3 & false & false & false & true \\ l_7 & false & W_{7,5} & false & W_{7,7} \end{array}\rangle,$$

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where
$W_{7,7}=$ "there is a token in place l_1",
$W_{2,6}=W_{7,5}=\neg W_{7,7}$.

If predicate $W_{7,7}=true$, then token γ stays in place l_7 without a new characteristic. Only when predicate $W_{7,7}=false$, i.e., when predicates $W_{2,6}=W_{7,6}=true$, token γ enter place l_5 without any characteristic, too. Token α_i enters place l_4 and obtains as a next characteristic an IM that we shall describe in more details. Having in mind that the i-th expert can use only a part of the criteria and can estimate only a part of the alternatives, we can construct the IM of his/her estimations in the form

$$S_i = \begin{array}{c|c} & A_{l_1}\ A_{l_2}\ \dots\ A_{l_{p_i}} \\ \hline C_{i_1} & \\ & \langle \alpha^i_{j,k} \beta^i_{j,k} \rangle \\ C_{i_2} & \\ & (1 \le j \le q_i \le q_{cu}, \\ \vdots & \\ & 1 \le k \le p_i \le p) \\ C_{i_{q_i}} & \end{array}$$

where: $\alpha^i_{j,k}, \beta^i_{j,k} \in [0,1],$, $\alpha^i_{j,k}+\beta^i_{j,k} \le 1$ and $\langle \alpha^i_{j,k}, \beta^i_{j,k} \rangle$ is the i-th expert estimation for the k-th alternative about the j-th criterion; $C_{i_1},...,C_{i_{q_i}}$ and $A_{l_1},...,A_{l_{p_i}}$ are only those of the criteria and alternatives which the i-th expert prefers. In the cases when pair $\langle \alpha^i_{j,k}, \beta^i_{j,k} \rangle$ does not exist, we will work with pair $\langle 0,1 \rangle$.

On the other hand, when all α-tokens transfered to place l_4, token β enters place l_6 with a characteristic, the set of all α-token's characteristic, i.e., "$\{S_1, S_2, ..., S_m\}$".

The GN-transition Z_2 has the form:

$$Z_2 = \langle \{l_4, l_5, l_8\}, \{l_8, l_9, l_{10}\}, \begin{array}{c|ccc} & l_8 & l_9 & l_{10} \\ \hline l_4 & true & false & false \\ l_5 & false & false & true \\ l_8 & false & W_{8,9} & false \end{array} \rangle,$$

where
$W_{8,9}=$ "there is a token in place l_{11}".

The α-tokens are collected without any characteristic in place l_8. They will continue their path to place l_9, without a characteristic, too, when there is an objective estimation of the alternatives (token δ in place l_{11}). Token γ obtains as a characteristic the following IM

$$S = \begin{array}{c|c} & A_1 \ A_2 \ \ldots \ A_p \\ \hline C_1 & \\ & \langle \alpha_{j,k}, \beta_{j,k} \rangle \\ C_2 & \\ & (1 \leq j \leq q_{cu}, \\ \vdots & \\ & 1 \leq k \leq p) \\ C_{q_{cu}} & \end{array}$$

where $\alpha_{j,k}$ and $\beta_{j,k}$ can be calculated by different formulas, with respect to some specific aims. For example, such formulas are the following:

$$\begin{cases} \alpha_{j,k} = \dfrac{\sum\limits_{i=1}^{m} \delta_i . \alpha_{j,k}^i}{m} \\ \beta_{j,k} = \dfrac{\sum\limits_{i=1}^{m} \varepsilon_i . \beta_{j,k}^i}{m} \end{cases}$$

(here the average degrees of experts' reliability are taken into account),

$$\begin{cases} \alpha_{j,k} = \dfrac{\sum\limits_{i=1}^{m} \delta_{i,j} . \alpha_{j,k}^i}{m} \\ \beta_{j,k} = \dfrac{\sum\limits_{i=1}^{m} \varepsilon_{i,j} . \beta_{j,k}^i}{m} \end{cases}$$

(here only the experts' degrees of reliability estimated by the corresponding criteria are taken into account).

$$\begin{cases} \alpha_{j,k} = \dfrac{\sum\limits_{i=1}^{m} \overline{\alpha}_{j,k}^i}{m} \\ \beta_{j,k} = \dfrac{\sum\limits_{i=1}^{m} \overline{\beta}_{j,k}^i}{m} \end{cases},$$

where $\overline{\alpha}_{j,k}^i$ and $\overline{\beta}_{j,k}^i$ can also be calculated by various formulas, according to particular goals and experts' knowledge. For example, such formulas can be:

$$\begin{cases} \overline{\alpha}_{j,k}^i = \gamma_i . \dfrac{\alpha_{j,k}^i . \delta_{i,j} + \beta_{j,k}^i . \varepsilon_{i,j}}{\gamma_i + 1} \\ \overline{\beta}_{j,k}^i = \gamma_i . \dfrac{\alpha_{j,k}^i . \varepsilon_{i,j} + \beta_{j,k}^i . \delta_{i,j}}{\gamma_i + 1} \end{cases}$$

or

$$\begin{cases} \overline{\alpha}^i_{j,k} = \alpha^i_{j,k}.\dfrac{\delta_{i,j}+1-\varepsilon_{i,j}}{2} \\ \overline{\beta}^i_{j,k} = \beta^i_{j,k}.\dfrac{\varepsilon_{i,j}+1-\delta_{i,j}}{2} \end{cases}.$$

The first formula takes into account not only the rating of each expert by the different criteria, but also the number of times he has given an opinion (the first time is neglected, since he does not have a rating then). Obviously, the so constructed elements of the IM satisfy the inequality: $\alpha_{j,k} + \beta_{j,k} \leq 1$. This IM contains the average experts estimations taking into account experts ratings. Let each one of the criteria $C_j (1 \leq j \leq q_{cu})$ have a priority $\varphi_j \in [0,1]$. This information will be put in the initial characteristic of token β. We can determine for every alternative A_k the global estimation $\langle \alpha_k, \beta_k \rangle$, where

$$\begin{cases} \alpha_k = \dfrac{\sum\limits_{j=1}^{q_{cu}} \varphi_j.\alpha_{j,k}}{q_{cu}} \\ \beta_k = \dfrac{\sum\limits_{j=1}^{q_{cu}} \varphi_j.\beta_{j,k}}{q_{cu}} \end{cases}.$$

Transition Z_3 can be activated only when token δ enters place l_{11} with an initial (unique) characteristic in the form of an IM with elements (objective) values about the different criteria:

$$T = \begin{array}{c|c} & A_1 \; A_2 \; \dots \; A_p \\ \hline C_1 & \\ & \langle a_{j,k}, b_{j,k} \rangle \\ \vdots & (1 \leq j \leq q_{cu}, \\ & 1 \leq k \leq p) \\ C_{q_{cu}} & \end{array}$$

where: $a_{j,k}, b_{j,k} \in [0,1]$ and $a_{j,k} + b_{j,k} \leq 1$.

Here we mention a significant difference between the four examples. It constitutes in the time needed for the experts to understand how well they have made their evaluations and prognoses. In the case of election prognoses, the experts obtain their own score at the moment of the final announcement of the results of the vote. On the other hand, when a job candidate is evaluated, the experts can estimate his/her work in the company in a longer period of time, including periods of adaptation and training, and first finished projects. Moreover, in such case the expert's appraisal may be subjective and liable to refutation.

The transitions Z_2 and Z_3 will be active until all α-tokens from place l_8 go to place l_{12} through place l_9. Its form is

$$Z_3 =< \{l_9, l_{10}, l_{11}\}, \{l_{12}, l_{13}, l_{14}\},$$

$$\begin{array}{c|ccc} & l_{12} & l_{13} & l_{14} \\ \hline l_9 & true & false & false \\ l_{10} & false & false & true \\ l_{11} & false & true & false \end{array}.$$

In the present model, token δ leaves the GN via place l_{13} without a final characteristic, but in the future GN-models they will obtain characteristics related, e.g. to the behaviour of the process flow and GN functioning, to the alternatives and criteria, etc.

Token α_i enters place l_{12} with final characteristic *"expert's new rating,* $\langle \delta_i, \varepsilon_i \rangle$*, and new number of participances in expert investigations,* γ_i'*".* The values of this characteristic are estimated by formulas

$$\gamma_i' = \gamma_i + 1,$$

and

$$\begin{cases} \delta_i' = \dfrac{\gamma_i.\delta_i + \dfrac{c_M - c_i}{2}}{\gamma_i'}, \\ \varepsilon_i' = \dfrac{\gamma_i.\varepsilon_i - \dfrac{c_M - c_i}{2}}{\gamma_i'}, \end{cases},$$

where

$$c_i = \frac{\sum\limits_{j=1}^{q_{cu}} \sum\limits_{k=1}^{p} ((\alpha_{j,k} - a_{j,k})^2 + (\beta_{j,k} - b_{j,k})^2)^{1/2}}{pq_{cu}},$$

and

$$c_M = \frac{\sum\limits_{i=1}^{n} c_i}{n}.$$

Other formulas for the expert's rating are also possible and they will be discussed in a next research.

Token γ enters place l_{14} with a characteristic "IM T". We remind that its previous characteristic is IM S.

The GN-transition X_1 (the first of the new transitions) has the form:

$$X_1 = \langle \{l_6, l_{14}\}, \{k_9, k_{10}\}, \begin{array}{c|cc} & k_9 & k_{10} \\ \hline l_6 & true & false \\ l_{14} & false & true \end{array} \rangle.$$

Token γ from l_{14} enters place k_{10} with a characteristic "list of all used criteria in the process of decision making". Therefore, it contains the criteria, given to the experts, as well as the new criteria, introduced by the separate experts and used in the time of the process.

Token β from l_6 enters place k_9 with a characteristic "IM U", where

$$U = \begin{array}{c|c} & A_{1,1} \ldots A_{1,m} A_{1,m+1} A_{1,m+2} \ldots A_{p,1} \ldots A_{p,m} A_{p,m+1} A_{p,m+2} \\ \hline C_1 & \\ & \langle \alpha_{k,i,j}, \beta_{k,i,j} \rangle \\ \vdots & (1 \leq i \leq m, \\ & 1 \leq j \leq q_{cu}, \\ C_{q_{cu}} & 1 \leq k \leq p) \end{array},$$

where $\langle \alpha_{k,i,j}, \beta_{k,i,j} \rangle$, $\langle \alpha_{k,m+1,j}, \beta_{k,m+1,j} \rangle$ and $\langle \alpha_{k,m+2,j}, \beta_{k,m+2,j} \rangle$ are, respectively, the evaluations of the i-th expert $(1 \leq i \leq m)$, aggregated evaluation of all experts (with index $i = m+1$) and objective result (with index $i = m+2$) for the k-th alternative about j-th criterion. Alternatives $A_{k,i}$ for each i coincide, but their evaluations are different and by this reason we can interprete them as different objects. IM U can have simpler form, if we include in it only experts evaluations, i.e., without the aggregated evaluations and objective results.

Token ζ enters place k_{11} with initial characteristic "(meta)criterion for a choice for near criteria". We discuss its meaning below.

$$X_2 = \langle \{k_9, k_{11}\}, \{k_{12}\}, \begin{array}{c|c} & k_{12} \\ \hline k_9 & true \\ k_{14} & true \end{array} \rangle.$$

In place k_{12}, tokens β and ζ unite in token β with a characteristic "IM V, list of near pair of criteria". The first component of this characteristic is obtained by the procedure, that we describe following [5,6,8]. It is the basic component of the so called intercriteria analysis. Here, it is described from intuitionistic fuzzy point of view (see, [8]).

Let us have the set of objects $O = \{O_1, O_2, ..., O_n\}$ that must be evaluated by criteria from the set $C = \{C_1, C_2, ..., C_m\}$.

Let us have an IM

$$A = \begin{array}{c|ccccccc} & O_1 & \cdots & O_i & \cdots & O_j & \cdots & O_n \\ \hline C_1 & a_{C_1,O_1} & \cdots & a_{C_1,O_i} & \cdots & a_{C_1,O_j} & \cdots & a_{C_1,O_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ C_k & a_{C_k,O_1} & \cdots & a_{C_k,O_i} & \cdots & a_{C_k,O_j} & \cdots & a_{C_k,O_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ C_l & a_{C_l,O_1} & \cdots & a_{C_l,O_i} & \cdots & a_{C_l,O_j} & \cdots & a_{C_l,O_n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ C_m & a_{C_m,O_1} & \cdots & a_{C_m,O_i} & \cdots & a_{C_m,O_j} & \cdots & a_{C_m,O_n} \end{array},$$

where for every p, q $(1 \leq p \leq m,\ 1 \leq q \leq n)$:

(1) C_p is a criterion, taking part in the evaluation,
(2) O_q is an object, being evaluated.

(3) a_{C_p,O_q} is a variable, formula or $a_{C_p,O_q} = \langle \alpha_{C_p,O_q}, \beta_{C_p,O_q} \rangle$ is an intuitionistic fuzzy pair, that is comparable about relation R with the other a-objects, so that for each i, j, k: $R(a_{C_k,O_i}, a_{C_k,O_j})$ is defined. Let $\overline{R}$ be the dual relation of R in the sense that if R is satisfied, then $\overline{R}$ is not satisfied and vice versa. For example, if "R" is the relation "$<$", then $\overline{R}$ is the relation "$>$", and vice versa.

Let $S^{\mu}_{k,l}$ be the number of cases in which

$$\langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle \leq \langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle$$

and

$$\langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle \leq \langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle,$$

or

$$\langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle \geq \langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle$$

and

$$\langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle \geq \langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle$$

are simultaneously satisfied.

Let $S^{\nu}_{k,l}$ be the number of cases in which

$$\langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle \geq \langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle$$

and

$$\langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle \leq \langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle,$$

or

$$\langle \alpha_{C_k,O_i}, \beta_{C_k,O_i} \rangle \leq \langle \alpha_{C_k,O_j}, \beta_{C_k,O_j} \rangle$$

and

$$\langle \alpha_{C_l,O_i}, \beta_{C_l,O_i} \rangle \geq \langle \alpha_{C_l,O_j}, \beta_{C_l,O_j} \rangle$$

are simultaneously satisfied.

Obviously,

$$S^{\mu}_{k,l} + S^{\nu}_{k,l} \leq \frac{n(n-1)}{2}.$$

Now, for every k, l, such that $1 \leq k < l \leq m$ and for $n \geq 2$, we define

$$\mu_{C_k,C_l} = 2\frac{S^{\mu}_{k,l}}{n(n-1)}, \quad \nu_{C_k,C_l} = 2\frac{S^{\nu}_{k,l}}{n(n-1)}.$$

Hence,

$$\mu_{C_k,C_l} + \nu_{C_k,C_l} = 2\frac{S^{\mu}_{k,l}}{n(n-1)} + 2\frac{S^{\nu}_{k,l}}{n(n-1)} \leq 1.$$

Therefore, $\langle \mu_{C_k,C_l}, \nu_{C_k,C_l} \rangle$ is an IFP.

Now, we can construct the IM

$$\begin{array}{c|ccc} & C_1 & \cdots & C_m \\ \hline C_1 & \langle \mu_{C_1,C_1}, \nu_{C_1,C_1} \rangle & \cdots & \langle \mu_{C_1,C_m}, \nu_{C_1,C_m} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ C_m & \langle \mu_{C_m,C_1}, \nu_{C_m,C_1} \rangle & \cdots & \langle \mu_{C_m,C_m}, \nu_{C_m,C_m} \rangle \end{array},$$

that determines the degrees of correspondence between criteria $C_1, \ldots, C_m$.

When objects $O_1, ..., O_n$ and criteria $C_1, ..., C_m$ from the above procedure coincide with our alternatives and criteria, respectively, we obtain the IM V that is the first component of the token β characteristic. The second component of this characteristic is obtained on the basis of the token ζ characteristic. Using it, we obtain the list of the near criteria.

Token η enters place k_{13} with initial characteristic "(meta)criterion for a choice of a better criterion between two given ones". This (meta)criterion can determine the better criterion because it is easier for checking, chipper, requires less time for checking. etc.

$$X_3 = \langle \{k_{12}, k_{13}\}, \{k_{14}\}, \begin{array}{c|c} & k_{14} \\ \hline k_{12} & true \\ k_{13} & true \end{array} \rangle.$$

In place k_{14}, tokens β and η unite in token β with a characteristic "list of bad criteria".

$$X_4 = \langle \{k_{10}, k_{14}\}, \{k_{15}\}, \begin{array}{c|c} & k_{15} \\ \hline k_{10} & true \\ k_{14} & true \end{array} \rangle.$$

In place k_{15}, tokens β and γ unite in token β with a characteristic "list of good criteria", i.e., $x_{cu}^{\beta} = x_{cu}^{\gamma} - x_{cu-1}^{\beta}$, where operation "$-$" between both characteristics is in set-theoretical sense.

3 Conclusion

The paper is a first attempt to unite two mathematical objects - the GNs and intercriteria analysis, that is used for increasing the effectiveness of decision making processes.

In a next research we shall discuss a possible extensions of the so constructed GN-model including other decision making activities and other applications of the intercriteria analysis.

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